

The Relationship between Area and Order of Harmonics

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Abstract: In this paper, we explore the relationship between the area of the Chladni plate and the order of harmonics formed on said plates. This is done by carrying out an experiment that finds the frequencies at which the first, second, third, fourth and fifth frequencies form, and then by analyzing the data, the relationship between the area of the Chladni plate and the order of harmonics is found.

Keywords: Resonance, Area of the Chladni plate, Order of Harmonics, Resonance frequency, Distribution of sand.

I. INTRODUCTION

Cymatics, the study of patterns created by vibrations, examines how sound waves interacting with physical media can generate intricate and often aesthetically pleasing formations through wave interference (Babu, 2021 #1650). Various factors can influence the outcomes of cymatic experiments, including the size of the surface medium. Cymatics has applications in numerous fields: in acoustics, it aids in understanding sound wave behavior, which is crucial for designing speakers, musical instruments, and spaces with superior acoustic qualities; in materials science, it contributes to surface patterning and vibration analysis, enabling the creation of materials with specific textures and properties; and in art, it inspires the creation of visually captivating works and interactive installations that respond to sound, enriching sensory experiences. This extended essay investigates the research question: *How does increasing the area of the Chladni plate (900, 756.25, 625, 506.25, 400 cm²) affect the frequency (Hz) at which the distribution of harmonics is formed?* The focus is on understanding how the surface area impacts the frequency at which the first nodal pattern emerges, shedding light on the intricate relationship between sound and physical media.

II. BACKGROUND INFORMATION

Before conducting this experiment, it is essential to understand the fundamental principles of cymatics. These principles include:

Sound Waves:

Sound waves are longitudinal waves that propagate through a medium such as air, water, or solid materials. As these waves travel, they cause particles in the medium to oscillate, compressing and rarefying in a wave-like motion (Vasdev, 2020 #1640). This motion plays a crucial role in influencing wave behavior and the formation of patterns during the experiment.

Nodes and Antinodes:

When sound waves interact with a surface, distinct motion patterns emerge. Nodes are points on the surface where destructive interference causes minimal or no displacement. In contrast, antinodes are regions of maximum displacement resulting from constructive interference (Kumar, 2020 #1641). The frequency of the waves and the material properties of the medium determine the arrangement and number of nodes and antinodes. This experiment focuses on understanding how varying the surface area impacts the frequency at which the first and subsequent harmonics form.

Resonance:

Resonance occurs when an object vibrates at its natural frequency, amplifying oscillations. In cymatics, resonance causes the surface to produce clear nodal and anti-nodal patterns. These patterns depend on the physical characteristics of the surface, such as its geometry. Frequencies ranging from 30 to 400 Hz are applied in this study to observe fundamental and higher harmonics on Chladni plates, analyzing the relationship between surface area and resonance frequency (Miljković, 2021 #1644).

Young's Modulus:

Young's Modulus measures a material's stiffness, expressed as the ratio of stress to strain within its elastic limit. A high modulus indicates greater stiffness and resistance to deformation, while a low modulus suggests flexibility. This property helps determine the plates' ability to maintain structural integrity during vibrations.

Flexural Strength:

Flexural strength refers to a material's capacity to endure stress before bending or breaking. Higher flexural strength ensures that the Chladni plates remain stable during testing, enabling the formation of precise nodal patterns without distortion.

FR-4:

FR-4, a fiberglass-reinforced epoxy laminate, is commonly used in electronics due to its flame resistance and mechanical properties. Its density of approximately 1.85 g/cm³ allows it to vibrate effectively and create distinct resonance patterns. With a Young's Modulus of 22 GPa and a flexural strength of 415 MPa, FR-4 is sufficiently stiff and resistant to bending. Its damping properties further enhance the focus on target frequencies by minimizing extraneous vibrations.

III. ASSUMPTIONS

Several assumptions were made to ensure the reliability and consistency of this experiment:

Uniform Behavior of the Medium:

It is assumed that the FR-4 material maintains consistent density and uniform response across all frequencies, enabling valid comparisons of nodal patterns (Hristova-Vasileva, 2016 #1646).

Constant Amount and Distribution of Sand:

A uniform layer of sand is consistently applied to each plate to ensure that any observed changes are due to surface area variations, not inconsistencies in sand placement.

Stable Environmental Conditions:

Environmental factors, such as temperature and humidity, are assumed to remain constant throughout the experiment. Variations in these factors could impact results by altering the medium's properties.

Negligible Damping Effects:

Damping factors, including air resistance and friction, are assumed minimal to simplify resonance and nodal pattern analysis. While minor vibration amplitude reductions may occur, their effects are considered negligible in the controlled setup.

IV. VARIABLES

Independent variable: Area of the Chladni plate

The independent variable is the surface area of the Chladni plates. Plates of various dimensions (30x30, 27.5x27.5, 25x25, 22.5x22.5, and 20x20 cm²) are used to explore how area influences nodal pattern formation. Larger plates are expected to produce more complex patterns due to increased wave freedom, while smaller plates should exhibit simpler configurations (Shridhar, 2011).

Dependent variable: The frequency at which the first and subsequent harmonics appear

The dependent variable is the frequency at which the first and subsequent harmonics appear. This involves identifying the exact frequencies that produce specific nodal patterns by incrementally increasing the frequency and making precise adjustments (Worland, 2011).

Control variable:

Control variables include material composition, wave generation methods, and environmental conditions. All plates are made from uniform FR-4 material to isolate the effect of surface area. A continuous waveform generator ensures consistent amplitude and frequency adjustments, while temperature and humidity are controlled to maintain stability.

Uncontrolled variables:

Some factors remain uncontrolled and may introduce variability in results. These include slight weight differences between plates, variations in sand grain density, minor imperfections in plate surfaces, and environmental noise or vibrations. Although efforts were made to minimize these effects, they highlight the experiment's inherent limitations.

V. EXPERIMENTAL DESIGN

Apparatus

This Experiment uses the Arbor Scientific Mechanical Wave Generator Kit to produce the vibrations needed for this cymatics-based apparatus.

1. Sine wave generator
2. Mechanical wave driver
3. Power drill
4. Jigsaw
5. Screw and Allen(Hex) key of appropriate size
6. Sandpaper or Sander
7. Vice
8. Box
9. Tarp (or a Long plastic bag)
10. Dry Sand
11. Container
12. Ruler
13. Pencil
14. Medium(FR-4)
15. Hand broom
16. A computer for data recording

Setup

Drying the sand (Only do this if you have wet sand)

1. Collect wet sand from a nearby source
2. Place 2-3 dry tissues on top of each other to create a flat surface
3. Place and spread the wet sand on the tissues (make sure to cover as much area as possible)
4. Carefully place the tissues and sand into a microwave and set the timer to 3:30 seconds
5. After the timer goes off, open the microwave door and let it cool down for 10-15 seconds
6. Carefully take the tissue and sand and check to see if it has become dry or not
7. If It hasn't, repeat steps 4-6 as many times as needed
8. If it has, carefully move the sand from the tissue, into a container of some sort for easier us

Preparing the Medium(FR-4)(Only do this step if you do not already have a material that has the areas needed)

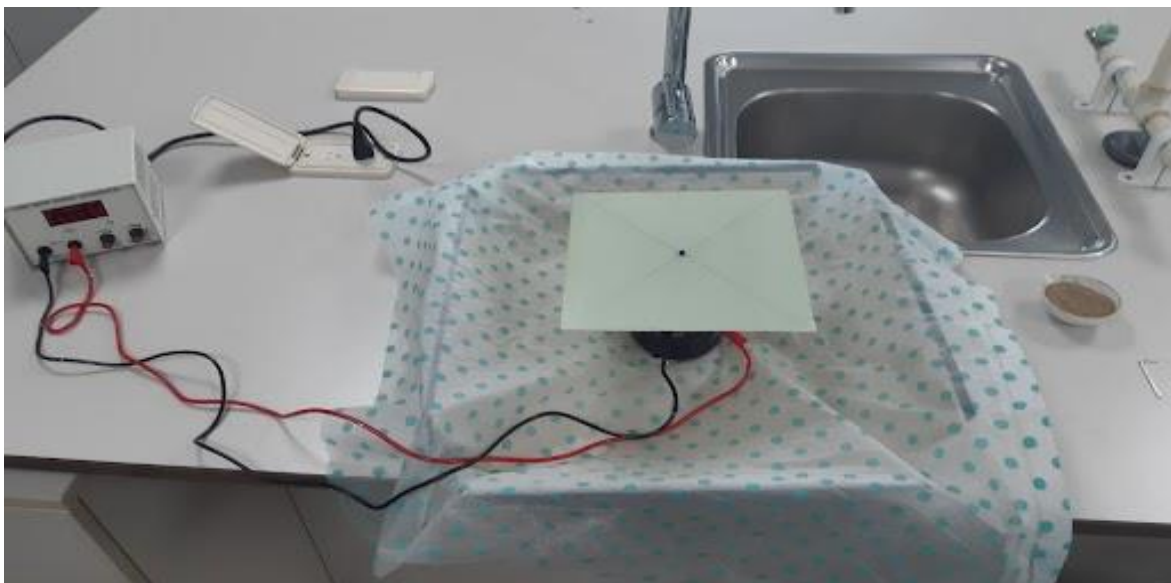
1. Mark each of the 5 boards with a pencil to outline their desired size. Make sure to draw the lines thickly so that they are evident.
2. Then within the desired sized squares, draw the diagonals(the intersection of the two diagonals is where you will drill a hole)
3. Using the vice properly pin the board to a flat surface, and make sure that the part that needs to be cut off is not under anything.
4. Begin using the jigsaw carefully to cut the medium into its desired size.
5. Using sandpaper or sander, clean the edges of the medium such that they are as straight and clean as possible.
6. Then place the medium vertically and hold it so that you can Drill a hole in it(make sure to drill a hole as close to the center as possible, otherwise it will be unbalanced when placed on the wave driver)

Sand collection

1. Cut the connection between each of the four walls of the box such that there are four “flaps”
2. Place the tarp on top of the box and spread it out so that the tarp forms a tiny inverse dome due to it curving upwards because of the flaps.

Data Collection Methodology

1. Plug the Sine wave generator into a nearby plug and place it near the setup.
2. Connect the sine wave generator to the Mechanical wave driver via the red and black wires
3. Place the mechanical wave diver into the center of the sand collection setup



4. Lock the driver's piston and place the Medium on top of the screw hole, and securely screw it to the wave generator.
5. When the medium is tightly in place unlock the driver and be careful not to push down on the medium as it will break the paper within the wave generator and render it useless
6. Sprinkle an adequate amount of Dry sand onto the medium.
7. Turn on the wave generator, and slowly raise the frequency.
8. When the sand starts moving rapidly to form a harmonic, try to pinpoint the exact frequency by increasing/decreasing the frequency and see the changes made in the harmonic.(If harmonic becomes more clear, go down and vice versa)

9. When the experiment is done, turn the generator off and tilt the wave generator to its side such that the sand falls into the tarp.
10. Place the wave generator outside of the tarp, and fold the tarp in such a manner that allows you to transport the sand within into the container once more.
11. After the sand is transported, place the tarp as it previously was collect any sand that was not caught by the tarp with a hand broom, and put it into the dry sand container.
12. Lock the mechanical wave driver, unscrew the medium, and then repeat steps 4-12 per each medium size.

Safety Considerations

Be careful with sand as it can get in your eye and cause irritation or even blindness

Be careful when using the power tools as if they aren't handled carefully, can result in severe injuries

Observations:

Top to bottom is the area of the plate in this order: 30x30, 27.5x27.5, 25x25, 22.5x22.5 and 20x20.

Left to right is the first, second, third, fourth, and fifth harmonic with the X meaning no harmonic was made.

Note: Some of the pattern photos are repeated as they were the clearest and of the highest photo quality.



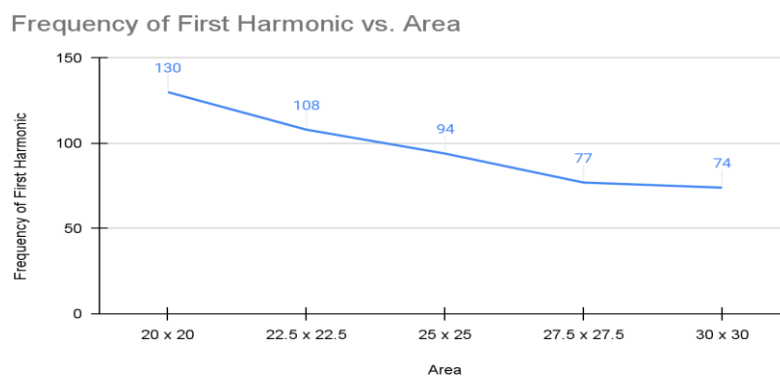
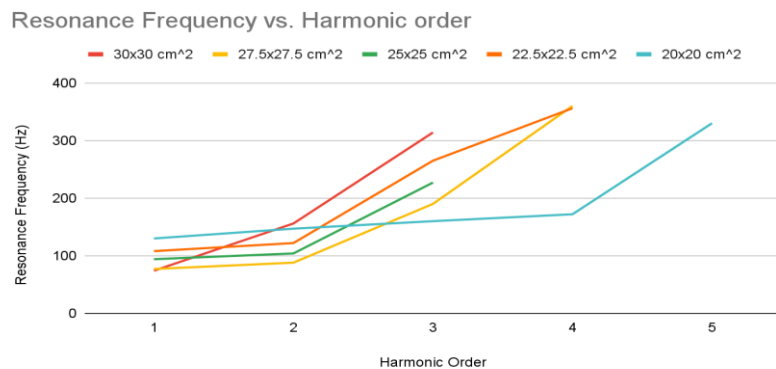
Data (Fig.1)

Area (cm ²)	First Harmonic (Hz)	Second harmonic (Hz)	3rd harmonic (Hz)	4th harmonic (Hz)	5th harmonic (Hz)
30x30	74 (oval)	156(center circle with quarter circles in each corner)	314 (center circle with half ovals on the tops of each side, the curved side facing the center circle)	X*	X*
27.5x27.5	77(horizontal parallel lines)	88(horizontal hyperbola)	190(center circle with quarter circles in each corner)	360(center circle with half ovals on the tops of each side, curved side is facing the center circle)	X*
25x25	94(2 vertically parallel lines)	104(vertical hyperbola)	227(center circle with quarter circles in each corner)	X*	X*
22.5x22.5	108(hyperbola)	122((oval but flipped 90 degrees)	265(center circle with quarter circles in each corner)	356(3 wavy lines)	X*
20x20	130 (oval)	147(2 parallel lines)	160(hyperbola)	172 (oval but flipped 90 degrees)	330(center circle with quarter circles in each corner)

*Indicates that there was no change from the previous harmonic or that the previous harmonic was only blurred/disturbed.

VI. RESULTS

According to the findings presented, the following graph has been made to showcase the frequency at which the first through fifth harmonics form for each manipulation and also a graph highlighting the relationship between the frequency of the first harmonic and the area of the medium.



VII. ANALYSIS

Pattern Recognition

When analyzing the cymatic patterns generated at various frequencies, it becomes evident that the plate size significantly influences the resulting nodal patterns, which grow increasingly intricate as the frequency rises. The first harmonic across all plate sizes generally produces simple geometric shapes, such as ovals, parallel lines, and hyperbolas. At higher frequencies, more elaborate patterns emerge, including consistent formations like a center circle accompanied by quarter circles or half ovals, which are observed as the third harmonic for all plate sizes. This suggests that pattern formation is influenced by both plate area and harmonic order (Meijer, 2021).

Interestingly, certain patterns recur regardless of plate size, highlighting potential resonance phenomena that transcend physical dimensions. For example, the third harmonic frequently features a central circle with quarter circles in each corner, indicating that specific frequencies may naturally resonate with the material. This could imply inherent resonant frequency patterns or suggest that certain frequencies are inherently more stable. Unique patterns also emerge based on material properties and dimensions; for instance, vertical parallel lines and ovals are exclusive to the 25x25 plate, while a three-wavy-line pattern appears only on the 22.5x22.5 plate. The asymmetry in the three-wavy-line pattern hints at intrinsic flaws in the plate or uncontrolled experimental variables.

Variations and disruptions in higher-frequency harmonics—where outlines become indistinct or unstable—highlight limitations in material properties, such as density, elasticity, and damping. External factors, including temperature, surface imperfections, and external vibrations, may also influence these results. This underscores the interplay between plate dimensions and material properties in determining harmonic patterns.

Analysis Based on Observations

The absence of new harmonic patterns at higher frequencies may indicate that the plate has reached its density threshold, beyond which it does not respond differently to additional energy. This plateau effect could stem from the material's intrinsic limitations or the thickness of the plate, restricting its ability to form higher-order harmonics.

Irregular patterns and disruptions point to medium instability, which may result from external vibrations, plate deformities, or inconsistencies in the excitation method (Shridhar, 2011). These factors suggest the onset of chaotic disorder at certain frequencies, emphasizing the experimental setup's constraints and the apparatus's limitations. Despite observable patterns, these irregularities reveal challenges in achieving consistency at the extremes of the setup's capabilities.

VIII. EVALUATION

The cymatics experiment demonstrated a clear relationship between harmonic frequencies, plate dimensions, and nodal patterns. Larger plates, such as the 30x30 cm size, displayed increasingly complex patterns at higher frequencies. Fundamental modes revealed basic oval shapes, while the second harmonic introduced more intricate patterns, including a circular base with quarter circles. Smaller plates exhibited distinct patterns, such as hyperbolas and inverted ovals, indicating variations in resonant properties.

The third harmonic consistently displayed a center circle with quarter circles for all plate dimensions, reflecting a resonance effect. However, diminishing pattern clarity at higher harmonics suggests plates nearing their resonance frequency. This plateau may be influenced by material properties, experimental configuration, or external conditions affecting both the material and the setup.

In summary, the data suggests an inverse relationship between the area of the medium and the frequency of the first harmonic. The frequency separation between successive harmonics increases with harmonic order. Furthermore, some higher-frequency patterns can emerge earlier when the medium's area is increased, as seen in the 30x30 cm plate. These findings align with previous studies but emphasize the importance of considering materials and experimental conditions.

Limitations

One significant limitation of this experiment is the reliance on a specific sine wave generator from the Arbor Scientific kit, restricting its accessibility. Attempts to create a functional drum and use a speaker as a frequency generator were unsuccessful due to inadequate elasticity and stiffness of improvised materials. Ultimately, the Arbor Scientific frequency generator provided by a physics teacher was essential for successful execution.

Additionally, sand displacement during the experiment reduced result reliability. For example, the oval pattern in the photo collage visibly contains uneven sand distribution. Off-center holes in the plates caused imbalances, introducing variability

in control conditions. Non-uniform sand density, visible as "smoky" regions in some photos, further contributed to inconsistencies.



IX. CONCLUSION

This cymatics experiment validated theoretical hypotheses through observed nodal patterns, which varied with plate dimensions and harmonic frequencies. Patterns demonstrated dependency on plate size, but certain frequencies consistently produced similar configurations, such as the third harmonic's central circle with quarter circles. This indicates that specific frequencies dictate pattern formation, irrespective of plate dimensions.

The inverse relationship between medium area and first harmonic frequency was evident, while higher harmonics exhibited a direct relationship with medium area. However, inconsistencies attributed to experimental limitations were noted. Higher frequencies led to pattern distortions or cessation, reflecting material constraints and external influences.

In conclusion, this study elucidates the interplay between harmonic frequencies and physical dimensions, offering insights with potential applications in material science and acoustic engineering.

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